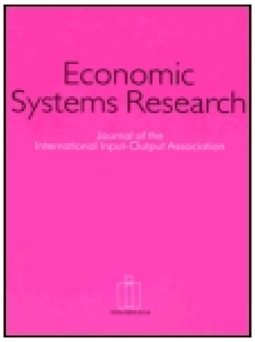

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Manfred Lenzen, Daniel D. Moran, Arne Geschke & Keiichiro Kanemoto

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A NON-SIGN-PRESERVING RAS VARIANT

MANFRED LENZEN*, DANIEL D. MORAN, ARNE GESCHKE
and KEIICHIRO KANEMOTO

ISA, School of Physics, The University of Sydney, Sydney, Australia

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We have developed a variant of the RAS generalised iterative scaling method that is able to change the sign between successive iterates, and thus fulfil constraints that are infeasible for existing RAS variants. Like earlier RAS variants, our method can handle constraints on arbitrarily sized and shaped subsets of matrix elements, include reliability of the initial estimate and the external constraints, and deal with negative values.

Keywords: Matrix balancing; RAS; Sign preservation

1. INTRODUCTION

It is a common feature of variants of the well-known RAS method that the signs of matrix elements in the initial estimate are preserved in the adjusted solution. In this respect, RAS differs from other constrained optimisation methods used for balancing input–output tables or Social Accounting Matrices, such as linear and quadratic programming, which do not preserve signs.

Researchers and statistical agency officers often prefer RAS methods over linear and quadratic programming approaches for updating input–output tables, because of RAS' computational simplicity and resulting ease of implementation. However, the sign-preserving behaviour presents quite an undesirable drawback. Consider, for example, the categories 'changes in inventories', 'taxes less subsidies on products', or 'taxes less subsidies on production'.[1] In practice, compilers of input–output tables are faced with situations in which superior data on these categories, either referring to individual table elements, or sub-sums of elements, are subject to sign flips between consecutive accounting years.

For example, the United Nations' Official Country database (UNSD, 2011) states changes in inventories of the Italian mining sector changed from 474,897,793€ in 2007 to 192,966,867€ in 2008. Assume that (a) the 2008 data point was used for updating a 2007 Italian input–output table to 2008, (b) this table distinguished a mining sector, (c) the prior matrix correctly represented Italian mining with negative changes in inventories in 2007, and (d) no other information on changes in inventories in 2008 existed. In this case, one would need to utilise the UN changes-in-inventories data point for 2008 in determining a

*Corresponding author. E-mail: m.lenzen@physics.usyd.edu.au

[1] As André Lemelin (personal communication 18 August 2011) points out, quantities such as net international investment position, net change in assets, and net change in liabilities, albeit more stable, are also subject to sign changes.

RAS multiplicand, to be applied to the 2007 Italian input–output table. This RAS multiplicand can only be positive however, making it impossible to change the negative sign of the 2007 changes-in-inventories value.

Similar issues hold for the taxes-less-subsidies row, for example when from one year to the following, gross subsidies become larger than gross taxes, or vice versa. If, for example, the initial estimate specified a taxes-less-subsidies element or a sub-sum of elements, with a negative sign in a particular year, and superior data prescribed a positive value for the following year, existing RAS variants would be unable to alter this sign during the adjustment process. At most, they would set the respective table element to zero. Thus, these RAS variants are prone to producing unrealistic outcomes for potentially sign-changing table categories, or may even lead to imperfect table balances.

With respect to changes in inventories, statisticians sometimes estimate gross capital formation (the sum of gross-fixed capital formation and changes in inventories) using the standard (G)RAS method. Then, using a separate estimate of gross-fixed capital formation, changes in inventories can be derived as a residual. This technique allows for changes in the signs of the changes in inventories, but obviously it requires a fair degree of complete information. In cases where this information does not exist, any RAS variant will produce an unrealistic updated table.

Existing RAS methods use prior-year information for updating a table, and in doing so they implicitly assume a structural relationship of transaction values over time. It is not clear how changes in inventories or taxes less subsidies in one year are related to their corresponding values in the subsequent year (Abramovitz, 1950; Blinder and Fischer, 1988), and consequently the RAS method may lack economic meaning. However, given that RAS variants are in widespread use, we take a pragmatic view and do not question RAS's appropriateness in this paper. In this sense, our contribution is similar to that by Temurshoev et al. (2013), who in their Equations 9a and 9b propose a mathematical solution to a particular problem in GRAS, which is one of the cases listed in our Table 1. Like the proposal by Temurshoev et al. (2013), our modification to RAS is purely mechanical and is not based on any economic theory. We suggest a mathematical strategy that will repair a particular shortcoming, and thus at least avoids unrealistic outcomes in particular situations.

We therefore present a modified generalised iterative scaling method that is able to change the sign between successive iterates, and thus fulfil constraints that are infeasible for existing RAS variants. We achieve this capability by introducing an additional adjustment step into the existing RAS procedure. Like earlier RAS variants, our method can handle constraints on arbitrarily sized and shaped subsets of matrix elements, include reliability of the initial estimate and the external constraints, and deal with negative values.

In what follows, we will first revisit an earlier publication (Lenzen et al., 2009), and recapitulate existing RAS approaches and point out those steps that cause the sign-preserving behaviour. We then introduce an additional computational step that enables RAS to adjust successive iterates in a way that they can comply with superior data that impose sign-changing constraints on the balancing task. Without loss of generality, we will cast our description in terms of the GRAS (Günlük-Senesen and Bates, 1988; Junius and Oosterhaven, 2003) and KRAS (Lenzen et al., 2009) methods. Nevertheless, some modifications to the basic RAS method should be achievable for any entropy function to be minimised (for example the minimum-information-loss function proposed by Lemelin, 2009). Finally, we will provide a real-world example, demonstrating the improved outcomes resulting from our modifications.

2. METHODOLOGY

2.1. Existing RAS Variants[2]

The RAS method – in its basic form – bi-proportionally scales a matrix $\mathbf{A}_0$ of unbalanced preliminary estimates of an unknown real matrix $\mathbf{A}$, using $\mathbf{A}$'s known row and column sums. The balancing process is usually aborted when the discrepancy between the row and column sums of $\mathbf{A}_0$ and $\mathbf{A}$ is less than a previously fixed threshold. Bacharach (1970) has analysed the bi-proportional-constrained matrix problem in great detail, in particular with regard to the economic meaning of bi-proportional change, the existence and uniqueness of the iterative RAS solution, its properties of minimisation of a distance metric, and uncertainty associated with errors in row and column sum data and with the assumption of bi-proportionality. The origins of the method go back several decades. Stone and Brown (1962), Bacharach (1970) and Polenske (1997) provide a historical background.

Over the years, Bacharach's original RAS approach has undergone many developments. The 'modified RAS' (MRAS) approach (Paelinck and Waelbroeck, 1963; Allen, 1974; Lecomber, 1975a) was developed for cases when some of the matrix elements of $\mathbf{A}$ are known in addition to its row and column sums. Oosterhaven et al. (1986) add constraints on aggregates of table elements to the standard row and column sum constraints. Similarly, Jackson and Comer (1993) use partition coefficients for groups of cells of a disaggregated base year matrix to disaggregate cells in an updated but aggregated matrix. Batten and Martellato (1985, pp. 52–55) discuss further constraint structures, involving intermediate and final demand data. Gilchrist and St Louis (1999; 2004) propose a three-stage 'TRAS' for the case when aggregation rules exist under which the partial aggregated information $\mathbf{A}^G$ can be constructed from its disaggregated form $\mathbf{A}$. Cole (1992) describes the general TRAS type that accepts constrained subsets of any size or shape. Gilchrist and St Louis, as well as Lenzen et al. (2006) demonstrate that the inclusion of partial aggregated information into the RAS procedure leads to superior outcomes.

Another variant of the MRAS method takes into account the uncertainty of the preliminary estimates, and contains the occurrence of perfectly known elements as a special case (Lecomber, 1975a; 1975b, with case studies in Allen, 1974, and Allen and Lecomber, 1975). Lahr (2001) takes into account the uncertainties of external constraints in treating the tolerances of the RAS termination criteria as functions of the varying reliabilities of row and column sums. Dalgaard and Gysting (2004) incorporate information about the reliability of external constraints (again row and column totals) into the balancing process as 'confidence factors'. Junius and Oosterhaven (2003) derive a generalised RAS ('GRAS') algorithm that can balance negative elements, by splitting the matrix $\mathbf{A}$ into positive and negative parts $\mathbf{P}$ and $\mathbf{N}$.

Lenzen et al. (2009) develop KRAS, a GRAS variant that works for conflicting external information and inconsistent constraints, under which previous RAS variants did not converge. KRAS combines features of many previous developments, such as constraints on subsets of table elements of arbitrary shapes, incorporation of reliability and uncertainty information, non-unity constraint coefficients, and negative table elements.

[2] Adapted from Lenzen et al. (2009).

Despite being far from exhaustive, this brief review of the history of the RAS method may suffice to show that negative elements became a concern only recently, and – to our knowledge – sign preservation has so far not been questioned at all.[3]

2.2. Sign-Preservation in RAS

Bacharach (1970, pp. 79–86) shows that the simple bi-proportional RAS algorithm can be derived from minimising

$$f(\mathbf{A},\mathbf{A}_0)=\sum_{i,j}A_{ij}\ \ln\frac{A_{ij}}{eA_{0ij}},\tag{1}$$

subject to constraints **u** and **v** on known row and column totals

$$\sum_j A_{ij}=u_i \quad \text{and} \quad \sum_i A_{ij}=v_j,\tag{2}$$

where e is the basis of the natural logarithm.[4] The GRAS method is derived in the same way. However, the initial estimate $\mathbf{A}_0$ (which becomes the solution $\mathbf{A}^{(0)}$ at step zero) is split into positive and negative parts according to $\mathbf{A}^{(0)}=\mathbf{P}^{(0)}-\mathbf{N}^{(0)}$. **A** is then alternately row- and column-scaled using diagonal scaler matrices $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$, so that after the nth round of balancing, $\mathbf{A}^{(n)}=\hat{\mathbf{r}}^{(n-1)}\mathbf{P}^{(n-1)}\hat{\mathbf{s}}^{(n-1)}-[\hat{\mathbf{r}}^{(n-1)}]^{-1}\mathbf{N}^{(n-1)}[\hat{\mathbf{s}}^{(n-1)}]^{-1}$. Junius and Oosterhaven's GRAS derivation arrives at a second-order polynomial that defines scalers

$$\begin{aligned} r_i^{(n)}&=\frac{u_i+\sqrt{u_i^2+4\sum_j P_{ij}^{(n)}\sum_j N_{ij}^{(n)}}}{2\sum_j P_{ij}^{(n)}} \quad \text{with}\\ P_{ij}^{(n)}&=P_{ij}^{(n-1)}s_j^{(n-1)},\\ N_{ij}^{(n)}&=N_{ij}^{(n-1)}[s_j^{(n-1)}]^{-1} \quad \text{and} \quad s_j^{(n-1)}=\frac{v_j+\sqrt{v_j^2+4\sum_i P_{ij}^{(n-1)}\sum_i N_{ij}^{(n-1)}}}{2\sum_i P_{ij}^{(n-1)}}. \end{aligned}\tag{3}$$

Lenzen et al. (2009) generalise the GRAS formulation by incorporating constraints on arbitrary subsets of matrix elements (including GRAS row and column sums), expressed as $\mathbf{Ga}=\mathbf{c}$, where **a** is the vectorisation of **A** above, and where the elements a_j of **a** are the same as the elements A_{ij} of **A**, except that they are arranged in a column vector instead of a matrix. Similarly, the KRAS initial estimate $\mathbf{a}_0$ is the vectorisation of the conventional

[3] See Polenske (1997), de Mesnard (2004), Lahr and de Mesnard (2004), Huang et al. (2008), and Temurshoev et al. (2011) for overviews.

[4] See Lemelin (2009) for a more in-depth elaboration on objective functions for RAS and minimum information loss principles.

initial estimate $\mathbf{A}_0$. The KRAS minimisation problem is

$$\text{minimise} \quad f(\mathbf{a}, \mathbf{a}_0) = \sum_j |a_j| \ln \frac{a_j}{ea_{0j}} \quad \text{subject to } \mathbf{Ga} = \mathbf{c}. \tag{4}$$

For N_C constraints, Equation 4 can be generalised to

$$r^{(n)} = \frac{c_i + \sqrt{c_i^2 + 4\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} \sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}}}{2\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)}} \quad \text{and}$$

$$a_j^{(n)} = a_j^{(n-1)}[r^{(n)}]^{\mathrm{Sgn}(a_j^{(n-1)}G_{ij})} \quad \text{with } i = n \mod N_C. \tag{5}$$

In Equation 5, the negative elements in Equation 3 have been replaced with negative coefficients on positive elements, but otherwise the formulation is exactly the same. There is only one scaler r_i for each constraint i, and these scalers are applied consecutively for all $i = 1, \ldots, N_C$. The r_i and a_j are calculated alternately. The KRAS feature of scaling negative elements by the inverse of the positive scaler is evident in the exponent $\mathrm{Sgn}(a_j^{(n-1)}G_{ij})$ in Equation 5. The mod operator denotes the modulo function, where a modulo b yield the remainder after division of a by b.

Equations 3 and 5 clearly show that scalers $r^{(n)}$ and $s^{(n)}$ are always positive, with the consequence that the sign of $a_j^{(n)}$ is always equal to the sign of $a_j^{(n-1)}$. This is the feature we are going to adjust in order to allow RAS to change the sign of iterates $a_j^{(n)}$.

2.3. A Non-Sign-Preserving RAS Variant

Assume for the time being a simple one-line constraint $Ga = c$, where $G = 1$, and assume that the initial estimate a_0, and hence also a, are initially positive, but that c is negative. This would apply for example to a situation where prior-year changes in inventories are positive (a_0, and then the first RAS iterate a), but negative (constraint c) for the current year. Equation 5 shows that in this case $\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)} = 0$, and $r^{(n)} = 0$. All that existing RAS variants can do is set the respective table element a to zero, but they cannot make it negative, as desired. Therefore, such a constraint is RAS-infeasible. Similarly, assume that $G = 1$, and the initial estimate a^0 and the first iterate a are negative, but c is positive. This is the situation of changes in inventories for Italian mining, which are positive in 2007 (a_0, and then the first RAS iterate a), but negative (constraint c based on UN data) in 2008. In this case, we find that $\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} = 0$, and the KRAS scaler $r^{(n)}$ is not even defined, because again the constraint is RAS-infeasible. Again, no RAS variant can make a positive as required. In Table 1, we define KRAS scalers for eight possible cases for varying signs of Ga and c. Four of these cases require a sign flip in a.

The scalers in Table 1 cannot be analytically derived, but their form can be motivated by four arguments:

(a) Columns 3 and 4 in Table 1 show that in every case requiring a sign flip, either $\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} = 0$ or $\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)} = 0$, and hence the term $4\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} \sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}$ in Equation 5 is always zero in these cases, leaving the term $r^{(n)} = (c_i + \sqrt{c_i^2})/2\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)}$ to work with.

TABLE 1. KRAS scalers for eight combinations of positive and negative G, a, and c.

G_{ij}	$a_j^{(n-1)}$	$\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)}$	$\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}$	c_i	Sign flip	$r^{(n)}$	$a_j^{(n)}$
1	1	>0	$=0$	2	No	$r^{(n)} = \frac{c_i+\sqrt{c_i^2}}{2\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)}}$	2
−1	−1	>0	$=0$	2	No		−2
1	−1	$=0$	>0	−2	No	$r^{(n)} = -\frac{c_i-\sqrt{c_i^2}}{2\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}}$	−2
−1	1	$=0$	>0	−2	No		2
1	1	>0	$=0$	−2	Yes	$r^{(n)} = \frac{c_i-\sqrt{c_i^2}}{2\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)}}$	−2
−1	−1	>0	$=0$	−2	Yes		2
1	−1	$=0$	>0	2	Yes	$r^{(n)} = -\frac{c_i+\sqrt{c_i^2}}{2\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}}$	2
−1	1	$=0$	>0	2	Yes		−2

Note: The combinations are characterised in Columns 1–5. Column 6 indicates whether a sign flip is required or not. Column 7 specifies a scaler that will achieve the desired iterate of a listed in the final Column 8. Note that the final two cases are equivalent to Equations 9a and 9b in Temurshoev et al. (2013).

Note by editor Bart Los: This table existed in its present form in the original submission dating back to 2011. Temurshoev et al. (2013) was first submitted in 2012. The present manuscript was published after Temurshoev et al. (2013) as a consequence of delays in its evaluation and processing. The two studies were developed independent of each other.

(b) Since the denominator may never be zero, we must use $2(\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} + \sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)})$ instead.

(c) The numerator term $\sqrt{c_i^2}$ would always produce the absolute $|c_i|$ of the constraint value c_i, which is one of the causes of the inability of conventional RAS techniques to facilitate a sign flip. Hence, we replace this term by c_i.

(d) The entire scaler $r^{(n)}$ needs to be sensitive to a mismatch of the signs of G and a, and hence we introduce a factor $\mathrm{Sgn}(\sum_j G_{ij}a_j^{(n-1)})$.

The various scalers listed in Table 2 can then be written as

$$r^{(n)} = \mathrm{Sgn}\left(\sum_i G_{ij}a_j^{(n-1)}\right) \frac{c_i + c_i}{2\left(\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} + \sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}\right)}$$

$$= \frac{\mathrm{Sgn}\left(\sum_j G_{ij}a_j^{(n-1)}\right) c_i}{\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} - \sum_{j,a_j^{(n-1)}G_{ij}<0} G_{ij}a_j^{(n-1)}} \quad (6)$$

The reader can verify that Equation 6 will reproduce all scalers in Table 1. Note that Table 1 does not list the trivial case where $\sum_{j,a_j^{(n-1)}G_{ij}>0} G_{ij}a_j^{(n-1)} > 0$ and $\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)} > 0$, because in this case, the conventional approach in Equation 5 applies.

Note also that the cases in Table 1 with $\sum_{j,a_j^{(n-1)}G_{ij}<0} -G_{ij}a_j^{(n-1)}$ in the denominator are equivalent to Equations 9a and 9b in Temurshoev et al. (2013), applying to cases where only negative elements of $\mathbf{A}$ participate in a constraint.

Note further that Table 1 refers to cases wherein constraints are either (a) only positive elements added or negative elements subtracted, or (b) only positive elements subtracted or negative elements added. In practice, and to stay with the Italian mining sector example from the introduction, this means that in a situation where the Italian input–output table distinguished more than one mining sector, and all 2007 changes in inventories were negative, our RAS procedure would enact a sign-flip for all mining sub-sectors. A situation where positive and negative 2007 sub-sector changes in inventories existed could be handled by conventional GRAS and KRAS, and sign-flips of total changes in inventories could occur through different scaling of the positive and negative table elements.

Note also that the implementation of this approach must ensure that sign-flip procedures are only applied to those elements that are allowed to undergo sign changes, for example changes in inventories and taxes less subsidies. In practice, this can be achieved in ways common to many optimisation problems, for example by setting up two additional a-sized vectors l and u containing lower and upper bounds for each element in a. Elements with $[l, u] = [0, \infty]$ or $[l, u] = [-\infty, 0]$ would then be excluded from any sign flips. Such exclusions can be realised computationally by simple 'if' queries and conditional statements.

During the review process of this paper, one referee asked whether Equation 6 can be derived as the result of a full-fledged, theory-based optimisation problem. The scalers listed in Table 1 do not differ much from the standard KRAS scaler in Equation 5, indeed only in that they adjust the signs of elements a_j. Looking at this feature from the perspective of optimisation theory, the sign-flip scalers essentially change the initial estimate in a way that all elements in $\mathbf{a}$ conform to the signs of constraints. They do not alter the optimisation behaviour of the method. Once all signs have been adjusted so that the iterates of $\mathbf{a}$ do not conflict anymore with constraints, optimisation proceeds in the usual way, using ordinary KRAS scalers as in Equation 5, based on the standard, theory-based optimisation principle.

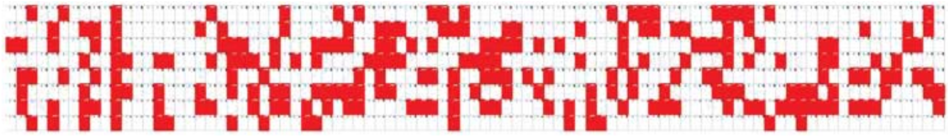
In other words, our approach does not define a new optimisation principle in order to enable sign changes. Instead, it alters the initial estimate in order to create a new reference point that enables the standard optimisation procedures to address constraints that previously were infeasible simply because of an incorrect sign. Reverting to our example in Table 1: if an initial estimate contained an element with a value of 1, and if this element were constrained by a value of −2, then the first iteration of the algorithm would see the sign-flip scalers alter the initial estimate by changing the respective element from 1 into −2. This first iterate $\mathbf{a}^{(1)}$ would then become the de facto initial estimate from which conventional theory-based optimisation would proceed. The solution of the procedure proposed in this work is hence optimal, given pre-imposed sign changes.

Of course, such a modification of the initial estimate could in principle be undertaken by the statistician prior to balancing, in a manual fashion. However, when dealing with large volumes of data, as well as complex constraints on arbitrarily shaped sub-aggregates of the table to be balanced, a manual intervention may not be practical.

3. APPLICATION

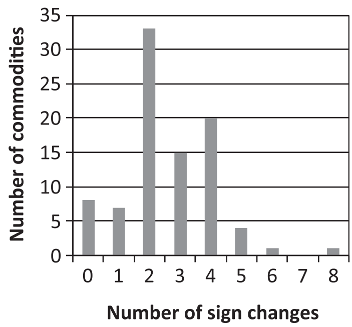
In order to demonstrate the relevance of sign changes, we examined a 2000–2008 time series of supply-use tables for Brazil (IBGE, 2011). In particular, we counted the number of instances of sign changes in the final demand category 'changes in inventories', which are

FIGURE 1. Instances of sign changes in the 'changes-in-inventories' category (goods only) of the Brazilian supply-use tables between 2000 and 2008.



Note: Each of the eight rows in the grid denotes a pair of years, starting with 2000–2001 in the top row, and ending with 2007–2008 in the bottom row. The columns represent the 89 goods. Each red field indicates a sign change.

FIGURE 2. Frequency distribution of sign changes across years.



reversals of inventory trends (Figure 1). For the years 2000–2008, this category distinguishes 110 commodities, amongst which are 89 goods and 21 services.

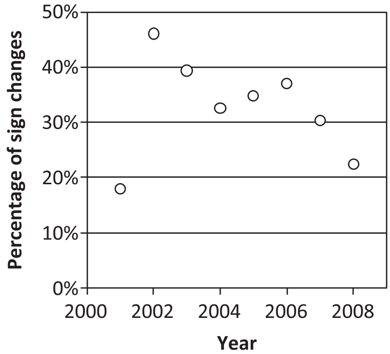
Most goods are affected by reversals in the trend of inventories (Figure 1). This is understandable, since otherwise stocks would grow or dwindle continuously. However, the frequency of such trend reversals is surprisingly high (Figure 2). Most commodities reverse stock trends at least twice in eight years (corresponding to well-known four-year business cycles, see Kitchin, 1923), and one commodity (coffee beans) showed continuously alternating stock levels in all eight years (possibly due to fluctuating climatic conditions and harvest outcomes, see the appendix).

In every year between 2000 and 2008, at least 20% and mostly more than 30% of all goods underwent reversals in stock trends (Figure 3).

We evaluated the performance of the sign-flip RAS variant against a conventional GRAS updating outcome. We took the 2000 Brazilian Supply-Use Tables as the first set of tables in an updating sequence spanning the years 2000–2008. The 2001 table is an update of the 2000 table, the 2002 table is an update of the just updated 2001 table, and so on. In the update from 2000 to 2001, original 2001 supply-use data are used as constraints, in the update from 2001 to 2002, original 2002 data are used, and so on. The appendix lists the updating results for the 990 elements of the changes-in-inventories columns. While the sign-flip variant exactly represents the original values in all updates, conventional GRAS is affected by 245 errors (highlighted).

Whilst the Brazilian supply-use tables are in principle no yardstick for the occurrence of trend reversals in changes in inventories of other countries, they at least provide us with an indication that frequent sign changes are possible in this input–output category. In the

FIGURE 3. Percentage of goods (out of a total of 89) undergoing sign changes in the category 'changes in inventories', that is subsequent reversals in inventory trends.



end, such frequent sign changes make sense, given that stocks cannot accumulate or deplete continuously. For taxes less subsidies on production, we observed only one change in signs over the entire period, occurring in the industry sector 'agriculture and forestry', where a subsidy turned into a tax between 2005 and 2006. This is not surprising since a subsidy can persist, because it is not affected by physical constraints in the same way as accumulating stocks of goods are.

4. CONCLUSIONS

Certain input–output data such as changes in inventories and taxes less subsidies can change signs between subsequent years. This circumstance is so far not catered for by any of the existing variants of the RAS method that is commonly used for balancing input–output tables. However, any table updating exercise that involves such sign-changing data needs to be carried out using a balancing method that allows sign changes. We have developed and, using the example of KRAS, presented a modification that can be added to the GRAS and KRAS variants, rendering these capable of realising sign changes prescribed by superior information.

Of course, one could circumvent the entire problem of sign changes by avoiding any net quantities in input–output tables, and only ever publish gross quantities. In this strategy, one would aggregate changes in inventories with investments, and disaggregate taxes less subsidies into separate taxes and subsidies. However, the first measure brings about an undesired loss of detail, and the second measure is infeasible whenever separate data on taxes and subsidies do not exist.

Another possibility is to create mirror accounts by converting the 'taxes-less-subsidies' row into a 'net tax' row by deleting all negative entries, and placing those as positive entries into an additional column called 'net subsidies' within the final demand block. Similarly, one could delete all negative changes-in-inventories elements, place them into an additional row called 'net decreases in inventories' within the primary inputs block, and relabel the column as 'net increases in inventories'. The resulting system would only have positive entries. However, this strategy does not appear to be used in practice, perhaps because the categories that would have to be created in such an artificial way have limited economic meaning. This is because these mirror accounts would only reflect the differences between taxes and subsidies, and between decreases and increases in inventories, but not their real absolute values.

Hence, until gross accounting is put into practice, the modifications we propose will enable RAS to deliver more realistic input–output table outcomes, especially in categories such as changes in inventories and net taxes.

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Appendix

Table A1. Evaluation of the sign-flip method.

Industry	Sign-flip RAS (= original changes in inventories data)									GRAS solution for changes in inventories				245 balancing errors out of 990 elements				
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2000	2001	2002	2003	2004	2005	2006	2007	2008
Rice in the husk	242	54	50	286	977	119	-524	-787	-624	242	54	50	286	977	119	0	0	0
Maize	320	151	-400	1945	-359	-867	186	167	3623	320	151	0	1945	0	0	186	167	3623
Wheat and other grains	−28	193	97	907	-723	-771	-974	-216	671	-28	0	0	0	-723	-771	-974	-216	0
Sugar cane	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Soy beans	75	-31	723	3299	1440	134	-349	2652	3608	75	0	723	3299	1440	134	0	2652	3608
Other agricultural products	7	4	166	198	-55	-6	-63	-156	81	7	4	166	198	0	0	0	0	81
Cassava	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tobacco	-26	47	-75	29	52	-63	-154	63	106	-26	0	-75	0	0	-63	-154	0	0
Cotton	78	90	-184	-125	722	585	-158	213	61	78	90	0	0	722	585	0	213	61
Cirtus fruit	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Coffee	304	-848	165	-703	400	-591	395	-827	366	304	0	165	0	400	0	395	0	366
Forestry products	38	37	150	-34	-210	193	222	162	33	38	37	150	0	0	193	222	162	33
Beef cattle and other live animals	563	663	645	811	350	207	207	202	277	563	663	645	811	350	207	207	202	277
Raw milk	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Pigs	0	19	-3	106	110	-55	-10	-179	-34	0	19	0	106	110	0	0	0	0
Poultry	0	-8	-20	-15	-5	2	1	1	4	0	-8	-20	-15	-5	0	0	0	0
Eggs	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fishing and aquaculture	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude oil and natural gas extraction	468	149	-286	9	877	2085	1327	-587	-876	468	149	0	9	877	2085	1327	0	0
Iron ore mining	323	179	-117	-178	-770	325	1140	980	-900	323	179	0	0	0	325	1140	980	0
Coal mining	10	2	6	3	20	10	-5	213	370	10	2	6	3	20	10	0	213	370
Non-ferrous metal mining	-38	216	119	-117	239	-38	491	105	735	-38	0	0	-117	0	-38	0	0	0
Non-metal mining	104	255	54	248	736	-93	94	429	539	104	255	54	248	736	0	94	429	539
Abbatoirs and other meat processing	392	202	-69	180	151	-299	-348	-514	-134	392	202	0	180	151	0	0	0	0
Pork	37	-67	-234	-169	92	242	113	51	244	37	0	0	0	92	242	113	51	244
Poultry meat	82	82	-439	39	18	-121	402	319	117	82	82	0	39	18	0	402	319	117
Processed seafood	26	51	-132	16	35	117	114	-64	92	26	51	0	16	35	117	114	0	92
Fruit and vegetable products	-201	-191	-739	-548	-78	131	763	204	269	-201	-191	-739	-548	-78	0	0	0	0
Soy products	281	-408	198	1208	1037	57	355	792	411	281	0	198	1208	1037	57	355	792	411
Oils and fats except maize oil	27	-38	-27	105	69	-50	19	4	178	27	0	0	105	69	0	19	4	178
Soy oil	4	4	-34	80	524	26	-80	-100	-32	4	4	0	80	524	26	0	0	0
Processed milk	-29	-42	-1	40	14	36	-49	-67	129	-29	-42	-1	0	0	0	-49	-67	0
Dairy products	280	15	-152	99	122	149	-159	90	294	280	15	0	99	122	149	0	90	294
Rice and rice products	194	229	-52	35	-215	-221	-57	-121	56	194	229	0	35	0	0	0	0	56
Wheat flour	50	132	46	276	59	-129	-235	-133	-323	50	132	46	276	59	0	0	0	0
Manioc flour	14	13	75	-42	62	-15	43	-34	-81	14	13	75	0	62	0	43	0	0
Maize oil and powders	74	196	313	-96	-217	50	-222	244	86	74	196	313	0	0	50	0	244	86
Refined sugar	370	494	41	1194	1237	-595	-1375	504	397	370	494	41	1194	1237	0	0	504	397
Roasted coffee	36	20	-107	-45	-63	-20	150	113	268	36	20	0	0	0	0	150	113	268
Instant coffee	17	19	-65	-45	71	153	64	79	56	17	19	0	0	71	153	64	79	56
Other food products	489	255	-4	-39	540	1020	-4	-826	961	489	255	0	0	540	1020	0	0	961
Beverages	571	698	293	315	678	255	406	-56	436	571	698	293	315	678	255	406	0	436
Tobacco products	40	-80	57	-12	106	89	23	-281	-275	40	0	57	0	106	89	23	0	0
Cotton ginning and spinning	280	265	272	125	83	-599	-451	42	-355	280	265	272	125	83	0	0	42	0
Woven fabrics	326	304	309	-407	-8	180	556	271	315	326	304	309	0	0	180	556	271	315
Other textile products	240	228	-576	-677	117	-347	159	182	877	240	228	0	0	117	0	159	182	877
Clothing	975	475	-202	-674	350	-82	22	1074	1530	975	475	0	0	350	0	22	1074	1530
Leather products except shoes	150	111	55	-29	224	459	-15	43	237	150	111	55	0	224	459	0	43	237
Shoes	80	171	163	69	331	204	-53	216	482	80	171	163	69	331	204	0	216	482
Wood products except furniture	158	133	20	52	-600	-629	-632	-370	-58	158	133	20	52	0	0	0	0	0
Paper pulp	13	10	60	-257	-185	-192	1	216	187	13	10	60	0	0	0	1	216	187
Paper and paper products	430	213	-356	-301	295	232	546	785	964	430	213	0	0	295	232	546	785	964
Publishing	346	353	77	113	58	122	-797	-549	-431	346	353	77	113	58	122	0	0	0
Liquefied petroleum gas	78	-240	150	62	580	279	327	446	215	78	0	150	62	580	279	327	446	215
Automotive gasoline	-339	-234	-409	-100	-55	520	363	-242	-161	-339	-234	-409	-100	-55	0	0	-242	-161
Gasoálcool	0	1	-350	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Fuel oil	23	-147	-280	-45	-280	-191	-340	-150	-528	23	0	0	0	0	0	0	0	0
Automotive diesel oil	480	454	500	-122	1422	850	538	300	-350	480	454	500	0	1422	850	538	300	0
Other refinery products	423	252	-469	-564	-144	774	-137	511	1161	423	252	0	0	0	774	0	511	1161
Alcohol	-237	-26	-280	640	-669	-429	-490	-151	225	-237	-26	-280	0	-669	-429	-490	-151	0
Inorganic chemical products	172	106	520	134	121	-782	-545	2490	-562	172	106	520	134	121	0	0	2490	0
Organic chemical products	37	51	-655	-501	-103	-474	-163	668	-940	37	51	0	0	0	0	0	668	0
Resins and elastomers	368	234	-16	341	1271	201	977	1036	1397	368	234	0	341	1271	201	977	1036	1397
Pharmaceutical products	269	463	-159	-124	484	117	4	-1432	-84	269	463	0	0	484	117	4	0	0
Fertilisers and pesticides	126	44	27	85	1223	-600	-725	1479	1369	126	44	27	85	1223	0	0	1479	1369
Soap, detergent and other toiletry products	305	384	208	195	697	398	-75	-168	-1122	305	384	208	195	697	398	0	0	0
Inks, varnishes and lacquers	104	33	17	60	-221	-131	269	229	371	104	33	17	60	0	0	269	229	371
Other chemical products	7	257	35	-166	900	379	-234	-475	640	7	257	35	0	900	379	0	0	640
Rubber products	231	151	-150	156	216	-539	158	-468	365	231	151	0	156	216	0	158	0	365
Plastic products	179	120	173	-541	-410	-1269	-345	-334	425	179	120	173	0	0	0	0	0	425
Cement	92	121	-338	-319	-110	23	434	-97	107	92	121	0	0	0	23	434	0	107
Other non-metallic mineral products	378	376	-650	-498	-547	-154	930	1309	2243	378	376	0	0	0	0	930	1309	2243
Pig iron	-6	4	30	1	3	192	325	500	-220	-6	0	0	0	0	0	0	0	-220
Structural steel products	264	673	-246	-204	-147	228	457	2054	6695	264	673	0	0	0	228	457	2054	6695
Non-ferrous metal products	69	137	-180	93	105	458	284	215	635	69	137	0	93	105	458	284	215	635
Cast iron	-1	5	-4	5	3	69	250	510	1226	-1	0	-4	0	0	0	0	0	0
Fabricated metal products	1199	1042	-31	378	796	1	35	736	1421	1199	1042	0	378	796	1	35	736	1421
Machines	261	696	-757	-126	182	-28	749	3042	4949	261	696	0	0	182	0	749	3042	4949
Household appliances	142	91	-127	575	307	-73	342	441	139	142	91	0	575	307	0	342	441	139
Office equipment	362	176	69	46	204	176	264	131	635	362	176	69	46	204	176	264	131	635
Electrical equipment	165	494	-338	-168	337	759	552	429	2253	165	494	0	0	337	759	552	429	2253
Electronic equipment	1268	867	-689	-364	1806	427	147	-609	-1077	1268	867	0	0	1806	427	147	0	0
Medical and optical equipment	334	76	-6	-10	-120	200	455	489	844	334	76	0	0	0	200	455	489	844
Vehicles	1131	9	203	-4	1541	1100	204	1600	3842	1131	9	203	0	1541	1100	204	1600	3842
Trucks and buses	37	24	12	1	-55	676	99	7	535	37	24	12	1	0	676	99	7	535
Vehicle parts	337	372	323	1019	3511	1744	-511	2980	4234	337	372	323	1019	3511	1744	0	2980	4234
Other transport equipment	173	675	1052	1516	-2184	-629	1644	1712	972	173	675	1052	1516	0	0	1644	1712	972
Furniture and other manufacturing	449	104	126	-346	443	50	365	-118	450	449	104	126	0	443	50	365	0	450
Scrap recycling	34	118	-2	2	2	18	7	5	386	34	118	0	2	2	18	7	5	386